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RESEARCH ARTICLE

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Vibrations of flexible pylons with time-dependent mass attachments under ground induced motions

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Abstract

This work examines how mass attachments that vary with time influence the motion of flexible pylons undergoing ground-induced vibrations. Specifically, an analytical solution is derived for a time-dependent, lumped mass attachment placed at the top of a cantilevered pylon undergoing time-harmonic longitudinal vibrations. At first, the solution to the governing equation of motion entails the recovery of the eigenvalue problem in the presence of a fixed mass. This is then followed by a generalization of this solution to allow for mass variability with time. It is noted that modal analysis no longer yields an uncoupled system of equations for the generalized coordinates, this necessitating a second step modal analysis, which is complicated by the fact that the system matrices are now time dependent. Results are presented in terms of time histories and frequency plots for the pylon displacement amplitude at the top, resulting from harmonic base motion of unit amplitude. The aim is to identify the frequency range over which the presence of a time-varying mass is either beneficial or detrimental in minimizing the pylon's kinematic response. Since the rate of change of the mass can be either positive or negative, the special cases of constant mass and of no mass can be recovered from the solution. Finally, this methodology can be generalized to include transverse and rotational vibrations of flexible pylons.

KEYWORDS
elastic waveguides, flexible beams, lumped mass, modal analysis, motion transmission, Pylons, structural vibrations, variable mass

1 | INTRODUCTION

The use of various types of beams, defined as unidimensional structural elements exhibiting flexure, torsion, and tension/compression is widespread in civil (columns, beams, etc.), mechanical (torsion bars, pylons, stringers, etc.) and aeronautical (blades, struts, etc.) engineering. A number of parameters enter the design of such beams, starting with the type and shape of the cross-section (hollow, non-uniform, etc.), type of construction (uniform, segmented, etc.), type of material

LIST OF SYMBOLS: c_P, c_S, k_P, k_S, Ω:, Pressure and shear wave velocities, pressure and shear wave numbers, forcing frequency; E, ρ, ν:, Modulus of elasticity, material density, Poisson's ratio; EI, EA, δ:, Flexural rigidity, axial rigidity, damping coefficient; L, D, d, A, I:, Beam length, mean diameter, thickness, cross-section area and moment of inertia; m, M, $m_L(t)$, μ:, Beam mass per length, beam total mass, attached lumped mass, mass rate; $N(x,t)$, $M(x,t)$, $Q(x,t)$:, Axial force, bending moment, shear force; R, $R(t)$:, Fixed mass ratio, time varying mass ratio; u, w, x_g:, Longitudinal displacement, transverse displacement, ground displacement; ω_n, Φ_n, q_n, m_n:, Eigenfrequencies, eigenfunctions, generalized coordinates, modal masses

